Math 127 **Ch 10 Review Worksheet** **Name**________________________

For #1 – 4, find the first 4 terms of each described sequence.

1) general term: $a_n = 2(3n-1)$

$a_n = 2(3n-1) = 6n - 2$

Start with a_1 and add 6 for to get each successive term:

$a_1 = 6(1) - 2 = 4$

So, the first 4 terms of the sequence are:

$$4,\ \ 4+1(6),\ \ 4+2(6),\ \ 4+3(6) \quad \Rightarrow \quad \mathbf{4, 10, 16, 22}$$

2) recursive formula: $a_1 = -2$ and $a_n = a_{n-1} - 3$ for $n \geq 2$

$a_n = a_{n-1} - 3$ indicates we need to subtract 3 for each successive term.

$a_1 = \mathbf{-2}$

$a_2 = a_1 - 3 = -2 - 3 = \mathbf{-5}$

$a_3 = a_2 - 3 = -5 - 3 = -8$

$a_4 = a_3 - 3 = -8 - 3 = -11$

So, the first 4 terms of the sequence are: $\mathbf{-2, -5, -8, -11}$

3) arithmetic sequence: $a_1 = 4; d = -1$

$a_1 = 4$

Start with a_1 and subtract 1 to get each successive term:

So, the first 4 terms of the sequence are:

$$4,\ \ 4+1(-1),\ \ 4+2(-1),\ \ 4+3(-1) \quad \Rightarrow \quad \mathbf{4, 3, 2, 1}$$

4) geometric sequence: $a_1 = 12; r = \frac{1}{3}$

The general term of a geometric sequence is: $a_n = a_1 \cdot r^{n-1}$

$a_1 = 12 \qquad r = \frac{1}{3}$

Start with a_1 and multiply by $\frac{1}{3}$ to get each successive term:

So, the first 4 terms of the sequence are:

$$12,\ \ 12\left(\frac{1}{3}\right) = 4,\ \ 4\left(\frac{1}{3}\right) = \frac{4}{3},\ \ \frac{4}{3}\left(\frac{1}{3}\right) = \frac{4}{9} \quad \Rightarrow \quad \mathbf{12, 4, \frac{4}{3}, \frac{4}{9}}$$

For #5 – 6, write a formula for the general term (the n^{th} term) of each described sequence. Then find a_9.

5) arithmetic sequence: $1, 5, 9, 13, 17, \ldots$

My method is different from what is typically taught.

I like to find $a_0 = a_1 - d$, the 0th term, which is not part of the sequence, yet is quite useful. a_0 is the constant term in the explicit formula for a_n and d is the multiplier of n. So, the explicit formula for an arithmetic sequence is always:

$a_n = a_0 + dn$ (note: you need to calculate a_0; it is not given)

For this sequence, $d = 5 - 1 = 4$, so $a_0 = a_1 - d = 1 - 4 = -3$

Then, the explicit formula is: $\boldsymbol{a_n = -3 + 4n}$

Finally, $\boldsymbol{a_9} = -3 + 4(9) = -3 + 36 = \mathbf{33}$

6) geometric sequence: $5, -10, 20, -40, \ldots$

The general term of a geometric sequence is: $a_n = a_1 \cdot r^{n-1}$

$a_1 = 5 \qquad r = \dfrac{-10}{5} = \dfrac{20}{-10} = -2$

Then, $\boldsymbol{a_n = 5 \cdot (-2)^{n-1}}$

Finally, $\boldsymbol{a_9} = 5(-2)^{9-1} = 5(-2)^8 = 5 \cdot 256 = \mathbf{1,280}$

For #7 – 11, find the indicated sum.

7) $\sum_{i=1}^{3} 4^i$

Method 1: Add 'em up:

$$\sum_{k=1}^{3} 4^k = 4^1 + 4^2 + 4^3 = 4 + 16 + 64 = \mathbf{84}$$

Method 2: Use the geometric series sum formula: $S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right)$

$a_1 = 4^1 = 4 \qquad r = 4 \qquad n = 3$

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 4 \cdot \left(\frac{4^3 - 1}{4 - 1}\right) = 4 \cdot \left(\frac{63}{3}\right) = 4 \cdot 21 = \mathbf{84}$$

8) Find $2 + 4 + 6 + 8 + \cdots$, the sum of the first 40 positive even integers.

Method 1: Think like Gauss

First, notice that $a_n = 2n$, so $a_{40} = 2(40) = 80$. Then:

$$S = \ \ 2 + \ \ 4 + \ \ 6 + \cdots + 80$$

$$S = 80 + 78 + 76 + \cdots + \ \ 2$$

$$2S = 82 + 82 + 82 + \cdots + 82 = 40(82)$$

Divide both sides by 2, to get

$$S = 20(82) = \mathbf{1,640}$$

Method 2: Use the arithmetic series sum formula: $S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n)$

Again, we need $a_{40} = 2(40) = 80$

$a_1 = 2 \qquad a_{40} = 80 \qquad n = 40$

$$S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n) = \left(\frac{40}{2}\right) \cdot (2 + 80) = 20(82) = \mathbf{1,640}$$

9) $\sum_{n=1}^{30} 4i$ (hint: this is arithmetic)

Method 1: Think like Gauss

First, we need $a_1 = 4(1) = 4$ and $a_{30} = 4(30) = 120$. Then:

$$S = \ \ 4 + \ \ 8 + \ \ 12 + \cdots + 120$$

$$S = 120 + 116 + 112 + \cdots + \ \ 4$$

$$2S = 124 + 124 + 124 + \cdots + 124 = 30(124)$$

Divide both sides by 2, to get

$$S = 15(124) = \mathbf{1,860}$$

Method 2: Use the arithmetic series sum formula $S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n)$

We need $a_1 = 4(1) = 4$ and $a_{30} = 4(30) = 120$

$a_1 = 4 \qquad a_{30} = 120 \qquad n = 30$

$$S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n) = \left(\frac{30}{2}\right) \cdot (4 + 120) = 15(124) = \mathbf{1,860}$$

10) Find the sum of the first 5 terms of this geometric sequence: $\frac{1}{3}, \frac{4}{3}, \frac{16}{3}, \ldots$

Method 1: Add 'em up (note that $r = 4$):

$$\frac{1}{3} + \frac{4}{3} + \frac{16}{3} + \frac{64}{3} + \frac{256}{3} = \mathbf{\frac{341}{3}}$$

Method 2: Use the geometric series sum formula: $S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right)$

$$a_1 = \frac{1}{3} \qquad r = 4 \qquad n = 5$$

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = \frac{1}{3}\left(\frac{4^5 - 1}{4 - 1}\right) = \frac{(1024 - 1)}{3 \cdot 3} = \frac{1023}{9} = \mathbf{\frac{341}{3}}$$

11) Find the sum of this infinite geometric series, if it exists: $3 - 1 + \frac{1}{3} - \frac{1}{9} + \cdots$

Method 1: Think like Gauss

$$a_1 = 3 \qquad r = -\frac{1}{3}$$

$$S = 3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \cdots$$

$$+\frac{1}{3}S = \quad 1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} - \cdots$$

$$\frac{4}{3}S = 3$$

Note that this series converges because: $|r| = \left|-\frac{1}{3}\right| = \frac{1}{3} < 1.$

Multiply both sides by $\frac{3}{4}$, to get

$$S = \frac{3}{4} \cdot 3 = \mathbf{\frac{9}{4}}$$

Method 2: Use the infinite geometric series sum formula: $S = a_1 \cdot \left(\frac{1}{1 - r}\right)$

$$a_1 = 3 \qquad r = -\frac{1}{3}$$

$$S = a_1 \cdot \left(\frac{1}{1 - r}\right) = 3\left(\frac{1}{1 - \left(-\frac{1}{3}\right)}\right) = \frac{3}{\frac{4}{3}} = \frac{3}{1} \cdot \frac{3}{4} = \mathbf{\frac{9}{4}}$$

For #12 – 14, simplify each factorial expression.

12) $\frac{(n+2)!}{n!}$

$$\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)n!}{n!} = \boldsymbol{(n+2)(n+1)}$$

13) $\frac{n+1}{(n+1)!}$

$$\frac{n+1}{(n+1)!} = \frac{n+1}{(n+1)\cdot n!} = \frac{\mathbf{1}}{\boldsymbol{n}!}$$

14) $\frac{(n+5)!}{(n+5)(n+4)(n+3)}$

$$\frac{(n+5)!}{(n+5)(n+4)(n+3)} = \frac{(n+5)(n+4)(n+3)(n+2)!}{(n+5)(n+4)(n+3)} = \boldsymbol{(n+2)!}$$

For #15 – 16, write each repeating decimal as a fraction in lowest terms.

15) $0.\overline{8}$

Let $x = 0.\overline{8}$. Then think like our old buddy, Gauss.

$$10x = 8.\overline{8}$$
$$-x = -0.\overline{8}$$

$$9x = 8$$
$$\boldsymbol{x = \frac{8}{9}}$$

16) $0.\overline{186}$

Let $x = 0.\overline{186}$. Then think like our old buddy, Gauss.

$$1000x = 186.\overline{186}$$
$$-x = -0.\overline{186}$$

$$999x = 186$$
$$x = \frac{186}{999} = \frac{\mathbf{62}}{\mathbf{333}}$$

Recall the rule for divisibility by 3: If the sum of its digits is divisible by 3, then the number is divisible by 3.

Same for 9: If the sum of its digits is divisible by 9, then the number is divisible by 9.

For #17 – 18, solve each problem. Round to the nearest dollar, unless otherwise specified.

17) Looking ahead to retirement, you sign up for automatic savings in a fixed-income 401K plan that pays 5% per year compounded annually. You plan to invest \$3500 at the end of each year for the next 15 years. How much will your account have in it at the end of 15 years?

Let's look at the series that results from this. Note: deposits are made at the **end** of the year.

The first year's deposit will earn 5% per year for 14 years.

The second year's deposit will earn 5% per year for 13 years.

…

The final year's deposit will earn 5% per year for 0 years.

Then, $S = 3{,}500 \cdot [(1.05)^{14} + (1.05)^{13} + \cdots + 1]$

Look at the series inside the brackets in reverse order:

$[1 + (1.05)^1 + (1.05)^2 + \cdots + (1.05)^{14}]$

$a_1 = 3{,}500 \qquad r = 1.05 \qquad n = 15$ years

Then,

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 3{,}500 \cdot \left(\frac{1.05^{15} - 1}{1.05 - 1}\right) \approx 3{,}500 \cdot \frac{1.07893}{0.05} = \mathbf{\$75{,}525}$$

18) Sergio deposits \$150 each month into an account paying annual interest of 6.5% compounded monthly. How much will his account have in it at the end of 5 years? How much interest will he have earned over the 5 years?

The answer to this question depends on when during the month Sergio deposits the \$150. The formula taught in class **assumes the deposit is made at the end of the month**. If that is true, then,

$$S = \frac{P\left[\left(1 + \frac{i}{n}\right)^{nt} - 1\right]}{\frac{i}{n}}$$

Note that I am using "i" for the annual interest rate because using "r" is confusing when relating this formula to other formulas.

$P =$ the amount of the monthly deposit ($P = 150$ in this problem)

$i =$ the annual interest rate ($i = .065$ in this problem)

$n =$ the interest compounding period ($n = 12$ for monthly compounding, 4 for quarterly, etc.)

$t =$ the number of years over which the interest accrues ($t = 5$ in this problem)

Then,

$$S = \frac{P\left[\left(1 + \frac{i}{n}\right)^{nt} - 1\right]}{\frac{i}{n}} = \frac{150\left[\left(1 + \frac{.065}{12}\right)^{12\cdot(5)} - 1\right]}{\frac{.065}{12}} = \mathbf{\$10{,}601}$$

Interest is the ending balance minus what Sergio deposited. Sergio deposited \$150 for 60 months.

$I = 10601 - 150(60) = \mathbf{\$1{,}601}$

For #19 – 22, solve each problem. Round to the nearest dollar, unless otherwise specified.

19) Lani invests \$225 each quarter in a fixed-interest mutual fund paying annual interest of 5% compounded quarterly. How much will her account have in it at the end of 6 years?

The answer to this question depends on when during the quarter Lani deposits the \$225. The formulas taught in class **assume the deposit is made at the end of the quarter**. If that is true, then,
using the approach from Problem 17:

5% annual interest, compounded quarterly is $\frac{5\%}{4} = 1.25\%$ per quarter.

The first quarter's deposit will earn 1.25% per quarter for 23 quarters.

The second quarter's deposit will earn 1.25% per quarter for 22 quarters.

…

The final quarter's deposit will earn 1.25% per quarter for 0 quarters.

Then, $S = 150 \cdot [(1.0125)^{23} + (1.0125)^{22} + \cdots + 1]$

Look at the series inside the brackets in reverse order:

$[1 + (1.0125)^1 + (1.0125)^2 \ldots + (1.0125)^{23}]$

$a_1 = 225 \qquad r = 1.0125 \qquad n = 24$ quarters

Then,

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) \;=\; 225 \cdot \left[\frac{(1.0125)^{24} - 1}{(1.0125) - 1}\right] \;\approx\; \$\mathbf{6,252}$$

20) A small business owner made \$50,000 the first year he owned his store and made an additional 9% over the previous year in each subsequent year. Find how much he made during his 4th year of the business. Also, find his total earnings during the first four years. Round to the nearest cent.

Method 1: Add 'em up

1st year: \$50,000

2nd year: $\$50{,}000 \cdot 1.09 = \$54{,}500$

3rd year: $\$50{,}000 \cdot 1.09^2 = \$59{,}405$

4th year: $\$50{,}000 \cdot 1.09^3 = \$\mathbf{64,751.45}$

Total: $\$(50{,}000 + 54{,}500 + 59{,}405 + 64{,}751.45) = \$\mathbf{228,656.45}$

Method 2: By formula,

$a_1 = 50{,}000 \qquad r = 1.09 \qquad n = 4$

4th year: $\$50{,}000 \cdot 1.09^{(4-1)} = \$50{,}000 \cdot 1.09^3 = \$\mathbf{64,751.45}$

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) \;=\; 50{,}000\left(\frac{1.09^4 - 1}{1.09 - 1}\right) \;=\; \$\mathbf{228,656.45}$$

21) A job pays a salary of \$34,000 the first year. During the next 8 years, the salary increases by 4% each year. What is the salary for the 9th year? What is the total salary over the 9-year period? Round to the nearest cent.

9th year salary:

$$a_9 = a_1 \cdot r^{(9-1)} = a_1 \cdot r^8$$

$$a_1 = 34{,}000 \qquad r = 1.04 \qquad n = 9$$

$$a_9 = 34{,}000 \cdot 1.04^8 = \mathbf{\$46,531.35}$$

Total salary over 9 years:

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 34{,}000\left(\frac{1.04^9 - 1}{1.04 - 1}\right) = \mathbf{\$359,815.04}$$

22) A hockey player signs a contract with a starting salary of \$810,000 per year and an annual increase of 6.5% each year, beginning in the 2nd year. What will the athlete's salary be, to the nearest dollar, in the 8th year?

8th year salary:

$$a_8 = a_1 \cdot r^{(8-1)} = a_1 \cdot r^7$$

$$a_1 = 810{,}000 \qquad r = 1.065 \qquad n = 8$$

$$a_8 = 810{,}000 \cdot 1.065^7 = \mathbf{\$1,258,729}$$

Total salary over 8 years (not part of the problem, but still fun):

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 810{,}000\left(\frac{1.065^8 - 1}{1.065 - 1}\right) = \mathbf{\$8,162,254}$$

Maybe it's worth losing a few teeth!

23) Evaluate the combination: $\binom{10}{5}$

$$\binom{10}{5} = \frac{10!}{5! \cdot (10-5)!} = \frac{10!}{5! \cdot 5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \mathbf{252}$$

24) Expand: $(2x-1)^5$

General Formula:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Step 1: Start with the binomial coefficients

$$\binom{5}{0} \quad + \binom{5}{1} \quad + \binom{5}{2} \quad + \binom{5}{3} \quad + \binom{5}{4} \quad + \binom{5}{5}$$

Step 2: Add in the powers of the first term of the binomial $(2x)$

$$\binom{5}{0}(2x)^5 \quad + \binom{5}{1}(2x)^4 \quad + \binom{5}{2}(2x)^3 \quad + \binom{5}{3}(2x)^2 \quad + \binom{5}{4}(2x)^1 \quad + \binom{5}{5}(2x)^0$$

Step 3: Add in the powers of the second term of the binomial (-1)

$$\binom{5}{0}(2x)^5(-1)^0 + \binom{5}{1}(2x)^4(-1)^1 + \binom{5}{2}(2x)^3(-1)^2 + \binom{5}{3}(2x)^2(-1)^3 + \binom{5}{4}(2x)^1(-1)^4 + \binom{5}{5}(2x)^0(-1)^5$$

Step 4: Simplify:

$$= (1)(32x^5)(1) + (5)(16x^4)(-1) + (10)(8x^3)(1) + (10)(4x^2)(-1) + (5)(2x)(1) + (1)(1)(-1)$$

$$= \mathbf{32x^5} - \mathbf{80x^4} + \mathbf{80x^3} - \mathbf{40x^2} + \mathbf{10x} - \mathbf{1}$$

Notice the following about a binomial expansion:

1. There are $(n+1)$ terms, where n is the exponent of the binomial being expanded.
2. n is the top number in every binomial coefficient.
3. The bottom numbers in the binomial coefficients count up from **0** to n.
4. If a term of the original binomial is negative, the signs in the solution alternate $+$ and $-$.
5. The exponent of the first term in the original binomial counts down from n to **0**.
6. The exponent of the second term in the original binomial counts up from **0** to n.
7. The exponents of the two terms in the original binomial add to n in every term of the expansion.

25) Find the 6^{th} term of the expansion of $(x^2 + y^4)^9$

General Formula:

$$(\boldsymbol{a} + \boldsymbol{b})^{\boldsymbol{n}} = \sum_{k=0}^{n} \binom{n}{k} \boldsymbol{a}^{\boldsymbol{n-k}} \boldsymbol{b}^{\boldsymbol{k}}$$

KEY POINT: Unfortunately, there are several ways to answer this question, based on how the "6^{th} term" is defined. In order to be consistent with the Pearson textbook and homework problems, we must set the value of k to be one less than the number of the term. Using this approach, the first term has $k = 0$, so the 6^{th} term has $k = 5$. Other sources name the terms differently, but we are concerned with how the solutions to tests and homework are handled in Washoe County.

The terms of the binomial expansion of $(a + b)^n$ are typically given by the formula:

$$\binom{n}{k} a^{n-k} b^k$$

Then, using the approach in the Pearson textbook for this problem:

$a = x^2 \qquad b = y^4 \qquad n = 9 \qquad term = 6 \qquad \text{Pearson textbook: } k = 5$

And, so,

$$\binom{n}{k} a^{n-k} b^k = \binom{9}{5} (x^2)^{9-5} (y^4)^5 = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} (x^2)^4 (y^4)^5 = \mathbf{126x^8y^{20}}$$

26) Find the common ratio of the geometric sequence: $80, 20, 5, \frac{5}{4}, \ldots$

In order to have a common ratio, the sequence must be geometric.

The common ratio can be found by dividing consecutive terms.

It is a good idea to do this twice to make sure the ratio between terms is the same.

$$r = \frac{a_2}{a_1} = \frac{20}{80} = \mathbf{\frac{1}{4}}$$

$$r = \frac{a_3}{a_2} = \frac{5}{20} = \mathbf{\frac{1}{4}} \quad ✓$$

27) Expand: $(3x + 2y)^6$

General Formula:

$$(\boldsymbol{a} + \boldsymbol{b})^{\boldsymbol{n}} = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Step 1: Start with the binomial coefficients

$$\binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}$$

Step 2: Add in the powers of the first term of the binomial $(2x)$

$$\binom{6}{0}(3x)^6 + \binom{6}{1}(3x)^5 + \binom{6}{2}(3x)^4 + \binom{6}{3}(3x)^3 + \binom{6}{4}(3x)^2 + \binom{6}{5}(3x)^1 + \binom{6}{6}(3x)^0$$

Step 3: Add in the powers of the second term of the binomial (-1)

$$\binom{6}{0}(3x)^6(2y)^0 + \binom{6}{1}(3x)^5(2y)^1 + \binom{6}{2}(3x)^4(2y)^2 + \binom{6}{3}(3x)^3(2y)^3 + \binom{6}{4}(3x)^2(2y)^4 + \binom{6}{5}(3x)^1(2y)^5 + \binom{6}{6}(3x)^0(2y)^6$$

Step 4: Simplify:

$$(1)(729x^6)(1) + (6)(243x^5)(2y) + (15)(81x^4)(4y^2) + (20)(27x^3)(8y^3) + (15)(9x^2)(16y^4) + (6)(3x)(32y^5) + (1)(1)(64y^6)$$

$$= \mathbf{729x^6 + 2916x^5y + 4860x^4y^2 + 4320x^3y^3 + 2160x^2y^4 + 576xy^5 + 64y^6}$$

Notice the following about a binomial expansion:

1. There are $(\boldsymbol{n} + \mathbf{1})$ terms, where $\boldsymbol{n}$ is the exponent of the binomial being expanded.
2. $\boldsymbol{n}$ is the top number in every binomial coefficient.
3. The bottom numbers in the binomial coefficients count up from **0** to $\boldsymbol{n}$.
4. If a term of the original binomial is negative, the signs in the solution alternate $+$ and $-$.
5. The exponent of the first term in the original binomial counts down from $\boldsymbol{n}$ to **0**.
6. The exponent of the second term in the original binomial counts up from **0** to $\boldsymbol{n}$.
7. The exponents of the two terms in the original binomial add to $\boldsymbol{n}$ in every term of the expansion.

28) Find the 8^{th} term of the expansion: $(4a-7b)^{10}$

General Formula:

$$(\boldsymbol{a}+\boldsymbol{b})^{\boldsymbol{n}} = \sum_{\boldsymbol{k}=0}^{\boldsymbol{n}} \binom{\boldsymbol{n}}{\boldsymbol{k}} \boldsymbol{a}^{\boldsymbol{n}-\boldsymbol{k}} \boldsymbol{b}^{\boldsymbol{k}}$$

KEY POINT: Unfortunately, there are several ways to answer this question, based on how the "8^{th} term" is defined. In order to be consistent with the Pearson textbook and homework problems, we must set the value of k to be one less than the number of the term. Using this approach, the first term has $k=0$, so the 8^{th} term has $k=7$. Other sources name the terms differently, but we are concerned with how the solutions to tests and homework are handled in Washoe County.

The terms of the binomial expansion of $(a+b)^n$ are typically given by the formula:

$$\binom{n}{k} a^{n-k} b^k$$

Then, using the approach in the Pearson textbook for this problem:

$a \Rightarrow 4a \qquad b \Rightarrow -7b \qquad n = 10 \qquad term = 8 \qquad \text{Pearson textbook: } k = 7$

And, so,

$$\binom{n}{k} a^{n-k} b^k = \binom{10}{7} (4a)^{10-7}(-7b)^7 = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1}(4a)^3(-7b)^7$$

$$= (-1)^7 \cdot 120 \cdot 64a^3 \cdot 823{,}543b^7 = \boldsymbol{6{,}324{,}810{,}240\, a^3 b^7}$$